ADDITIONAL MATHS

REVISION CHECKLIST

. Differentiation/Integration

. Algebra – indices, surds, quadratics, fractions

. Coordinate Geometry

. Volumes and Areas

. Factor and Remainder Theorem

. Trigonometry

DIFFERENTIATION + INTEGRATION

DIFFERENTIATION FROM FIRST PRINCIPLES

$$\frac{dy}{dy} = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

LEARN METHOD + PRACTISE THESE

DIFFERENTIATION USING RULE

$y = x^n \quad \frac{dy}{dx} = nx^{n-1}$

[POWER IN FRONT, SUBTRACT ONE FROM THE POWER]

TANGENTS TO CURVES

- FIND $\frac{dy}{dx}$
- FIND $\frac{dy}{dx}$ at the given pt. (sub in x value)
- USE $y - y_1 = m(x - x_1)$

STATIONARY POINTS (MAX/MIN)

- FIND $\frac{dy}{dx}$
- PUT $\frac{dy}{dx} = 0$ AND FIND x VALUES
- SUB x VALUES INTO EQUATION $y =$ TO FIND y VALUES
- FIND $\frac{d^2y}{dx^2}$ AND SUB IN x VALUES OF STATIONARY POINTS

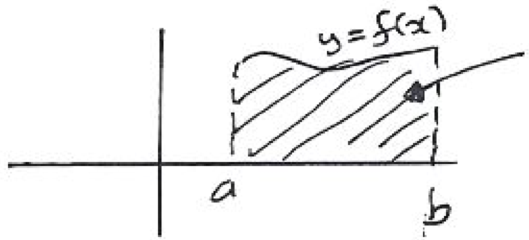
[IF $\frac{d^2y}{dx^2} > 0$ MIN IF $\frac{d^2y}{dx^2} < 0$ MAX]

INTEGRATION

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$$

[ADD ONE ONTO POWER, DIVIDE BY NEW POWER]

AREA UNDER CURVE



$$\int_a^b f(x) \, dx$$

TOP LIMIT − BOTTOM LIMIT

ALGEBRA

SURDS

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\sqrt{a} \times \sqrt{a} = a$$

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\frac{1}{(2-\sqrt{3})} = \frac{1}{(2-\sqrt{3})} \frac{\times(2+\sqrt{3})}{\times(2+\sqrt{3})} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

MULTIPLY BY WHATEVER'S IN THE DENOMINATOR WITH A CHANGE OF SIGN

INDICES

$$x^{p} \times x^{q} = x^{p+q}$$

$$\frac{x^{p}}{x^{q}} = x^{p-q}$$

$$(x^{p})^{q} = x^{pq}$$

INDICES

$$x^{-1} = \frac{1}{x} \qquad x^{-3} = \frac{1}{x^{3}}$$

$$x^{\frac{1}{2}} = \sqrt{x} \qquad x^{1/3} = \sqrt[3]{x}$$

$$x^{4/3} = \sqrt[3]{x^{4}}$$

$$x^{p/q} = \sqrt[q]{x^{p}}$$

ALGEBRAIC FRACTIONS

$$\frac{3}{2x+1} + \frac{4}{5x-3}$$

$$= \frac{3(5x-3)+4(2x+1)}{(2x+1)(5x-3)}$$

$$= \frac{15x-9+8x+4}{10x^{2}-6x+5x-3}$$

$$= \frac{7x-5}{10x^{2}-x-3}$$

QUADRATIC EQUATIONS

- **FACTORISING**

$$3x^{2}-10x+3=0$$

$$(3x-1)(x-3)=0$$

$$x=\tfrac{1}{3} \qquad x=3$$

- **FORMULA**

$$x = \frac{-b \pm \sqrt{b^{2}-4ac}}{2a}$$

- **COMPLETING THE SQUARE**

$$x^{2}-16x+61 = (x-8)^{2}-3$$

MIN VALUE $= -3$ WHEN $x=8$

CO-ORDINATE GEOMETRY

DISTANCE BETWEEN 2 POINTS

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

GRADIENT (m) $= \dfrac{\text{CHANGE IN } y}{\text{CHANGE IN } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

EQUATIONS OF LINES

$y = \underbrace{m}_{\text{gradient}}x + \underbrace{c}_{\text{y-intercept}}$

$y - y_1 = m(x - x_1)$ ← NEED GRADIENT AND A POINT

PARALLEL LINES – SAME GRADIENT

PERPENDICULAR LINES

$m_1 \times m_2 = -1$

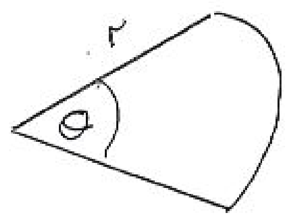
LINES AND CURVES INTERSECTING

SOLVE SIMULTANEOUSLY

SUB LINE INTO CURVE

VOLUMES + AREAS

ARCS + SECTORS



Area $= \dfrac{\theta}{360}(\pi r^2)$

Arc length $= \dfrac{\theta}{360}(\pi d)$

VOLUME OF CONE $= \frac{1}{3}\pi r^2 h$

CURVED SURFACE AREA $= \pi r l$

VOLUME OF SPHERE $= \frac{4}{3}\pi r^3$

SURFACE AREA $= 4\pi r^2$

Factor and Remainder Theorem

Remainder Theorem

If $f(x)$ is divided by $x-a$ the remainder $= f(a)$

eg Find the remainder when x^3+x-2 is divided by $x+2$

$f(x) = x^3 + x - 2$

$f(-2) = (-2)^3 + (-2) - 2$

$= -12$

remainder $= -12$

Factor Theorem

If $x-a$ is a factor of $f(x)$ then $f(a) = 0$

eg Show that $x-1$ is a factor of $x^3 - 7x + 6$

$f(x) = x^3 - 7x + 6$

$f(1) = (1)^3 - 7(1) + 6 = 0$

$\therefore$ $x-1$ is a factor

Factorising cubics

Once you have one factor you can long divide.

eg Show that $x-1$ is a factor of $2x^3-x^2-2x+1$ and hence factorise the expression.

$f(x) = 2x^3 - x^2 - 2x + 1$

$f(1) = 2(1)^3 - (1) - 2(1) + 1$

$= 0$

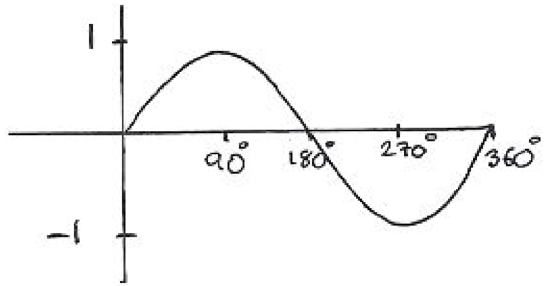
$\therefore$ $x-1$ is a factor

$$
\begin{array}{r|l}
 & 2x^2 + x - 1 \\
\hline
x-1 & 2x^3 - x^2 - 2x + 1 \\
 & 2x^3 - 2x^2 \ominus \\
\hline
 & \quad x^2 - 2x \\
 & \quad x^2 - x \ominus \\
\hline
 & \qquad -x + 1 \\
 & \qquad -x + 1 \\
\hline
 & \qquad 0 \quad 0
\end{array}
$$

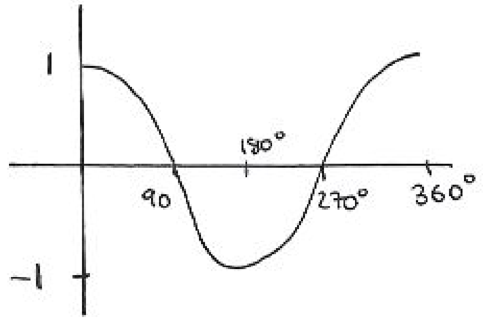
$2x^3 - x^2 - 2x + 1 = (x-1)(2x^2+x-1)$

$= (x-1)(2x-1)(x+1)$

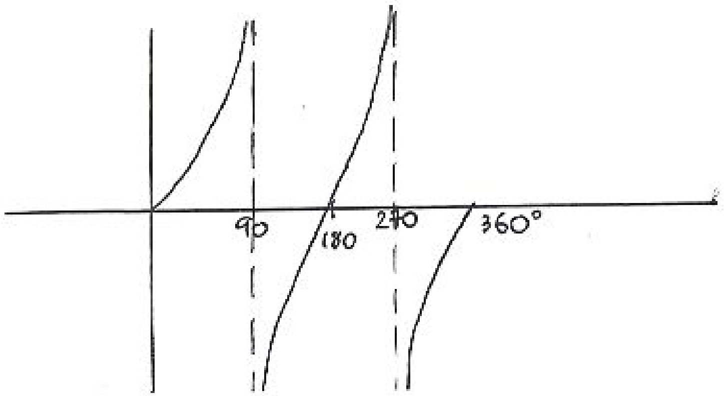
$y = \sin\theta$



$y = \cos\theta$



$y = \tan\theta$



TRIGONOMETRY

ANGLES IN 4 QUADRANTS

S	A
T	C

$180° - AAA$	AAA
$180° + AAA$	$360° - AAA$

SPECIAL ANGLES (YOU CAN USE CALCULATOR TO FIND THESE)

	0°	30°	45°	60°	90°
SIN	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
COS	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
TAN	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\to\infty$

SOLVING SIMPLE EQUATIONS

$3\cos\theta = 2$

$\cos\theta = \frac{2}{3}$

$AAA = \cos^{-1}\left(\frac{2}{3}\right) = 48.2°$

$\theta = 48.2$

$\theta = 360 - 48.2 = 311.8°$

$2\cos 3\theta = -1$ (for all angles of θ between 0° and 180°)

$\cos 3\theta = -\frac{1}{2}$

$AAA = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

ignore −

cos −ve in 2nd + 3rd quadrants

2nd $3\theta = 180 - 60 = 120°$ (also 480°)

+360°

3rd $3\theta = 180 + 60 = 240°$

$\theta = 40°$ or $120°$

DON'T FORGET ALL THE TRIG FROM GCSE

SOH CAH TOA

PYTHAGORAS' THEOREM

$a^2 = b^2 + c^2$

SINE RULE

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

COSINE RULE

$a^2 = b^2 + c^2 - 2bc\cos A$

IF THERE IS MORE THAN ONE TRIANGLE IN THE SHAPE, DRAW OUT AND LABEL THE ONE YOU ARE USING